# Discounted Cash Flow Methods for Urban Forestry: Standard and Specialized Formulas 

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## Introduction

Discounted cash flow (DCF) analysis (or the income approach) is a method of valuation often used in forests managed for timber production objectives to obtain the present value of cash flows, or the value in current day dollars considering interest. But it can be challenging to use in urban forestry and arboriculture, as the cash flows for benefits and costs from single trees or urban forests are difficult to determine, and the mathematical structure of DCF analysis is somewhat complicated.

There is a simple-to-use (and free) forestry valuation software package called FORVAL that can be applied to urban forestry situations that require DCF calculations. But like most valuation software packages, it requires that cash flows be input in one of a few standard structures (single sum, terminating annuity, perpetual annuity, or perpetual periodic series). These standard structures have rigid assumptions about the cash flow sequences; for example, a cash flow occurring each year and beginning at the end of the first year or a cash flow occurring periodically every x years and beginning at year x . Benefits (income) and costs in urban forest and tree valuation situations do not always occur in these structured patterns and standard DCF
formulas do not handle irregular cash flows well. This is another primary reason the income approach is often difficult to apply in these situations. This publication explains how to use the software package for standard urban forestry applications, but also shows how to handle the nonstandards cash flows that occur in arboriculture and urban forestry situations.

## Basic DCF Formulas

First, what are the standard situations? These are the situations that the program can handle without adjustment of the formulas on a routine basis. Recall that in valuation "year 0" represents "today" on a time line. The cash flows are, thus, being discounted to year zero. The variables in these formulas are $\mathrm{V}_{0}$ or the value at year 0 (present value), i or the interest rate expressed as a decimal, $n$ or the number of compounding periods (years), and $V_{n}$ or the value $n$ years in the future (future value). The examples below use a $5 \%$ interest rate $(i=0.05)$. The standard formulas are:

- Single-sum discounting. The basic formula in DCF analysis simply moves a cash flow from the future to "today" using a specified interest rate. For example, the future cost of $\$ 406.00$ incurred ninety years from today has a discounted value of $\$ 5.03$. The formula for determining this value is:

$$
V_{0}=\frac{V_{n}}{(1+i)^{n}}
$$

- Present value of a terminating cash flow series. Often uniform cash flows occur on an annual basis, but the series may end; for example, with the death of the tree. The annual payment or cost in this case is represented by a. For example, the present value of annual revenues of $\$ 10.00$ over 90 years is $\$ 197.52$. The basic formula to calculate this value is:

$$
V_{0}=a \frac{(1+i)^{n}-1}{i(1+i)^{n}}
$$

- Present value of a perpetual annual cash flow series. In some urban forestry situations (such as the creation of a conservation easement), the value of an annual cash flow may occur forever. For example, the present value of an annual perpetual cash flow of $\$ 100.00$ is $\$ 2,000.00$. The calculation of a perpetual annuity is simply:

$$
V_{0}=\frac{a}{i}
$$

- Present value of a terminating periodic cash flow series. In some cases, benefits or costs may have a regular magnitude, but occur on a periodic, not an annual basis. An example is flood mitigation benefits of $\$ 195.66$ that occur every 20 years over 80 years, producing a present value of $\$ 115.95$. The formula for calculation of a terminating periodic series where $n$ is the number of periods, $t$ is the length of each period, and $\varphi$ is the periodic cash flow (or, $\mathrm{t}=20, \mathrm{n}=4$, and $\varphi=\$ 195.66$ ) is

$$
V_{0}=\varphi \frac{(1+i)^{n t}-1}{\left[(1+i)^{t}-1\right](1+i)^{n t}}
$$

Note that this equation is not commonly found on standard financial software packages. However, the present value of a terminating cash flow series, commonly found in these packages, can be used to solve for this present value if the periodic cash flow is converted to an annual cash flow using the formula:

$$
a=\varphi \frac{i}{(1+i)^{t}-1}
$$

In the case of the example above, the periodic cash flow of $\$ 195.66$ is converted to an annual cash flow of $\$ 5.92$. The $\$ 5.92$ annual cash flow produces the same present value of $\$ 115.95$ (with a small rounding error).

- Present value of a perpetual periodic cash flow series (Urban Tree Site Value). Like the land expectation value (LEV) in forestry for timber production, a site value for an urban tree can also be calculated by compounding the value of the tree's cash flows to the end of its "rotation" (defined as its viable life on the site) and assessing this over a perpetual time frame. An example would be a net present value of all costs and revenues over an 80 -year life cycle on an urban site of $\$ 1,000$. On a perpetual basis, where $U T S V$ is the urban tree site value and $N P V$ is the net present value of all benefits and costs of the tree for one "rotation," the urban tree site value is $\$ 1,020.59$, calculated using the equation below.

$$
U T S V=\frac{N P V(1+i)^{n}}{(1+i)^{n}-1}
$$

## Specialized DCF Urban Forestry Formulas

The standard formulas described above are familiar components of DCF and the simple examples should suffice to illustrate proper use. The UTSV formula is somewhat specialized, but the concept is the same as the standard LEV calculation in forestry; the reference on LEV clearly explains that formula. The following specialized formulas account for problems in applying DCF analysis in arboriculture and urban forestry situations. These situations are geometrically increasing cash flows, delayed cash flows, and non-annual cash flows. Each formula is accompanied by a simple example with a figure that illustrates the cash flows.

Present Value of a Geometrically Increasing Cash Flow Series. In some situations, the benefits or costs may increase annually at some exponential rate (g), expressed as a decimal.. For example, a tree's ability to sequester carbon may increase a given rate per year. The present
value of these benefits or costs, where $g$ is the percentage rate of growth of the annuity and the remaining variables are as defined previously, is:

$$
V_{0}=\frac{a}{(i-g)}\left[1-\left(\frac{1+g}{1+i}\right)^{n}\right]
$$

Example: Consider carbon sequestration benefits that begin at $\$ 1.82$ per acre and increase at $4 \%$ annually over a lifespan of 90 years (Figure 1). In this example, a equals $\$ 1.82$, g equals 0.04 , and i equals 0.05 , and $n$ equals 90 . The present value of the cash flow series is $\$ 105.08$.


Figure 1. Geometrically increasing cash flow series, starting at $\mathbf{\$ 1 . 8 2}$ and increasing at 4\% annually, and ending at year 90.

Present Value of Minimum Size Delayed Annual Cash Flows. In some urban trees, annual cash flows may not occur until the tree reaches a certain minimum size. In fact, this calculation is very common in urban forestry applications, as many urban forest benefits rely on a certain crown size or structure more than a particular age or DBH. Examples include privacy benefits, sound barrier benefits, air quality, health, and recreation benefits, and energy cost savings. These crown size assets only occur once the tree has developed a mature crown. The initial formula, where $n_{a}$ is the number of years for which the cash flow occurs, and $n_{v}$ is the number of years before the cash flow begins, is:

$$
V_{0}=a \frac{(1+i)^{n_{a}}-1}{i(1+i)^{n_{a}}(1+i)^{n_{v}}}
$$

Example: energy savings of $\$ 13.67$ annually begin once a tree reaches 10 years of age and ends when the tree dies at age 90 (Figure 2). In this case, a equal's $\$ 13.67, \mathrm{n}_{\mathrm{a}}$ equals 81 , and $\mathrm{n}_{\mathrm{v}}$ equals 9. The present value of the energy savings is $\$ 172.85$. An illustration shows this is correct. Using the standard present value of a terminating annual cash flow series formula, the present value of a 90-year cash flow of $\$ 13.67$ is $\$ 270.01$ and the present value of a 9 -year cash flow is $\$ 97.16$. Since the first 9 years are the ones without a cash flow, subtracting $\$ 97.16$ from $\$ 270.01$ produces a present value of $\$ 172.85$.


Figure 2. Minimum size delayed annual cash flows of $\$ 13.67$, beginning at age 10 and ending at age 90.

Present Value of Minimum Size Delayed Periodic Cash Flows. This formula is similar to the last formula in that it also accounts for a delay in the cost or benefit and calculates a present value that is contingent upon the tree reaching a certain minimum size. However, cash flows in this case are periodic or non-annual. This present value, where $n_{a}{ }^{t}$ is the number of years for which the series occurs, $t$ is the length of the time period, and $n_{p}$ is the number of years the series is delayed is:

$$
V_{0}=a \frac{(1+i)^{n_{a} t}-1}{\left[(1+i)^{t}-1\right](1+i)^{n_{a} t}(1+i)^{n_{p}}}
$$

Example: Consider the "windbreak" ability of a tree in a windstorm. First, the tree would need to reach a minimum size to have windbreak ability and, second, the benefit would occur
periodically, not every year. Assume the presence of a large oak prevents the removal of shingles from a windstorm that occurs once every 10 years eliminating a cost (in time, labor, and materials) of $\$ 150$. Further assume the first windstorm occurred in year 15 , then every 10 years following until year 85 (Figure 3). In this case, a equals $\$ 150$, $\mathrm{n}_{\mathrm{v}}{ }^{\mathrm{t}}$ equals 80 , t equals 10 , and $\mathrm{n}_{\mathrm{p}}$ equals 5. The present value is this cash flow is $\$ 183.11$. An illustration shows this is correct. The present value of a terminating periodic series of $\$ 150$ every 10 years for 80 years is $\$ 233.70$. The standard period annuity would have a first payment at year 10 , instead of year 15 , so the $\$ 233.70$ needed to be discounted for five years to obtain the correct PV of $\$ 183.11$.


Figure 3. Minimum size delayed periodic cash flows of $\$ 150$, beginning at age 15, occurring every ten years until age 90 .

Present Value of Patterned Terminating Periodic Cash Flow Series. Urban trees may have several systematic, "stacked" cash flows; a cash flow of a smaller magnitude (the base series) may occur on a frequent basis, but necessitate a cash flow of a larger magnitude (the stacked series) on an infrequent basis. In this case the larger cash flow is stacked onto the pattern of the smaller cash flow. The present value of this pattern with two cash flows could be calculated with this equation:

$$
V_{0}=a_{1} \frac{(1+i)^{n_{1} t_{1}}-1}{\left[(1+i)^{t_{1}}-1\right](1+i)^{n_{1} t_{1}}}-\left(a_{2}-a_{1}\right) \frac{(1+i)^{n_{2} t_{2}}-1}{\left[(1+i)^{t_{2}}-1\right]\left((1+i)^{n_{2} t_{2}}\right.}
$$

where $\mathrm{a}_{1}$ is the cash flow of the base series, $a_{2}$ is the cash flow of the stacked series, $i$ is the interest rate, $n_{l}$ is number of years the base series occurs, $t_{l}$ is length of the time period for the base series, and $n_{2}$ is number of years the stacked series occurs, and $t_{2}$ is length of the time period for the stacked series. The equation is set up for costs. So in a case like pruning costs (or negative numbers) the two terms in the equation would be added together.

Example: Continuing the pruning example from above, assume pruning takes place every five years starting at age 5 and continuing to age 90 . Then every tenth year an additional $\$ 100$ of pruning is necessary. Every five years the cost is $\$ 160$ and every ten years the cost is $\$ 260$. Or, there is $\$ 100$ of stacked cost (Figure 4). Then, $\mathrm{a}_{1}$ equals $\$ 160, \mathrm{a}_{2}$ equals $\$ 260, \mathrm{n}_{1}$ equals $18, \mathrm{n}_{2}$ equals $9, \mathrm{t}_{1}$ equals 5 , and $\mathrm{t}_{2}$ equals 10 . The first term equals $\$ 571.94$ and the second term equals $\$ 157.04$. Since both are costs, they are added to produce a present value of $\$ 728.98$.


Figure 4. Patterned terminating periodic cash flow series of $\mathbf{\$ 1 6 0}$ every five years and an additional $\$ 100$ stacked on that series every tenth year.

## Present Value of Minimum Size Delayed Patterned Terminating Cash Flows

Like other benefits or costs that do not begin until a minimum tree size occurs, patterned terminating benefits or costs need be discounted back to year zero. A systematic pruning of a tree on two levels is an example of this calculation; the pruning example in Figure 4 would qualify, for example, if it did not begin until year 15 .

$$
V_{0}=a_{1} \frac{(1+i)^{n_{1} t_{1}}-1}{\left[(1+i)^{t_{1}}-1\right](1+i)^{n_{1} t_{1}}(1+i)^{n_{V 11}}}-\left(a_{2}-a_{1}\right) \frac{(1+i)^{n_{2} t_{2}}-1}{\left[(1+i)^{t_{2}}-1\right]\left((1+i)^{n_{2} t_{2}}(1+i)^{n_{v 2}}\right.}
$$

Where $a_{1}$ is the cash flow of the base series, $a_{2}$ is the cash flow of the stacked series, $n_{1}$ is number of years for which the base series occurs, $t_{l}$ is length of the time period for the base
series, and $n_{2}$ is number of years the stacked series occurs, $t_{2}$ is length of the time period for the stacked series, $n_{v 1}$ is number of years the base annuity is away from year zero, and $n_{v 2}$ is number of years the stacked annuity is away from year zero.

Example: Consider the stacked pruning example above where minor pruning takes place every five years and major pruning takes place every ten years. However, the pruning process does not begin until year 15 (Figure 5). Then, $\mathrm{a}_{1}$ equals $\$ 160$, $\mathrm{a}_{2}$ equals $\$ 260, \mathrm{n}_{1}$ equals $16, \mathrm{n}_{2}$ equals $8, \mathrm{t}_{1}$ equals $5, \mathrm{t}_{2}$ equals $10, n_{v 1}$ equals 10 and $n_{v 2}$ equals 10 .. The first term equals $\$ 348.35$ and the second term equals $\$ 95.65$. Since both are costs, they are added to produce a present value of $\$ 444.00$.


Figure 5. Minimum size delayed patterned terminating cash flow series of $\mathbf{\$ 1 6 0}$ every five years and an additional $\$ 100$ stacked on the series every ten years, beginning at year 15.

## Summary

This publication should serve two purposes for the arborist or urban forester desiring background on DCF analysis. The basic formulas used in DCF analysis are discussed with simple examples. These examples provide a convenient review of each basic formula and its use. Specialized formulas that apply to situations that are common in urban forestry and arboriculture are also presented. Again simple examples show proper application and detailed cash flows allow the reader to more easily understand the application. These specialized formulas can be adapted to standard valuation software for use in urban forestry situations. These formulas should make DCF analysis much easier for arborists and urban foresters.

## References

Bullard, S.H., T.J. Straka, and C. B. Landrum. 2011. FORVAL. Online Forestry Investment Calculations. Mississippi State University. http://www.cfr.msstate.edu/forval/.

Bullard, S.H. and T.J. Straka. 1998. Basic Concepts in Forest Valuation and Investment Analysis. Jackson, MS: Forestry Suppliers:

Davis, L.S., K.N. Johnson, P. Bettinger, and T.E. Howard. 2005. Forest Management: To Sustain Ecological, Economic, and Social Values. Long Grove, IL: Waveland Press, Inc. .

Davey Tree Expert Company. 2009. National Tree Benefit Calculator. Davey Tree Expert Company. http://www.davey.com/cms/cus/f94711556cbd4c7b/treecalculator.html.

Klemperer, W.D. 1996. Forest Resource Economics and Finance. New York: McGraw-Hill, Inc.
Martin, C.W., R.C. Maggio, and D.N. Appel.1989. Contributory Value of trees to residential property in Austin, Texas Metropolitan Area. Journal of Arboriculture 15:72-75.

McPherson, E.G. and J.R. Simpson. 2002. "A Comparison of Municipal Forest Benefits and Costs in Modesto and Santa Monica, California." Urban Forestry and Urban Greening 1:62-74.

Straka, T.J. 1991. Valuing stands of precommercial timber. Real Estate Review 21(2):92-96.
Straka, T.J. 1992. Determining the present value of non-annual cash flows. Real Estate Review 22(2):22-25.

Straka, T.J. 2007. Valuation of bare forestland and premerchantable timber stands in forestry appraisal. Journal of the American Society of Farm Managers and Rural Appraisers 70(1):142-146.

Straka, T.J. 2009. Appraising damaged timber. Journal of the American Society of Farm Managers and Rural Appraisers 72(1):51-60.

Straka, T.J., and S.H. Bullard. 1994. FORVAL-A computer software package for forestry and natural resources project valuation. Journal of Natural Resources and Life Sciences Education 23(1):51-55.

Straka, T.J., and S.H. Bullard. 1996. Land expectation value calculation in timberland valuation. Appraisal Journal 64(4):399-405.

Straka, T.J., and S.H. Bullard. 2002. FORVAL: Computer software package for forestry investment analysis. New Zealand Journal of Forestry 46(4):8-11.

Straka, T.J., and S.H. Bullard. 2006. An appraisal tool for valuing forest lands. Journal of the American Society of Farm Managers and Rural Appraisers 69(1):81-89.

Straka, T.J., and J.E. Hotvedt. 1985. Economic aspects of the forest regeneration delay decision. Southern Journal of Applied Forestry 9(2):91-94.

Tietenburg, T.H. and L. Lewis. 2008. Environmental and Natural Resource Economics. Reading: Addison-Wesley.

Wolf, K. 2004. What could we lose? The economic value of urban forests. Sixth Annual Canadian Forest Conference Proceedings. University of Washington 2:1-8.

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